

10. Find the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \rightarrow \infty} \frac{x^3}{\sqrt{x^6 + 9}}$$

Thinking:

Evaluating the Limits

I've been working on determining the limits for the function as x approaches both positive and negative infinity. I've successfully calculated the limit as x approaches positive infinity, arriving at a specific value. My next step is to carefully analyze the behavior of the function as x tends towards negative infinity to find that limit, too.

Solving the Infinity Limit

I'm revisiting the limit problem as x approaches infinity. Having solved it before, I recall the steps clearly. I'll divide the numerator and denominator by x^3 (the effective highest power). This leads to simplifying the expression inside the square root. I now have the limit of $1/\sqrt{1+9/x^6}$. As x grows, the term with x in the denominator approaches zero.

Confirming Negative Infinity Limit

I've worked through the calculations for the limit as x approaches negative infinity, carefully accounting for the behavior of the cube root. The denominator now looks like $-\sqrt{1+9/x^6}$. As x approaches negative infinity, the fraction

with x approaches zero, leaving me with -1 . I'm satisfied that this is the correct answer. I am now ready to present the solution.

Reviewing and Finalizing

I have carefully reviewed the limits for positive and negative infinity, ensuring the correct approach and calculations. My analysis confirms the limit as x approaches positive infinity is 1. When x approaches negative infinity, the limit is -1. I have prepared a detailed explanation for both cases, ready for presentation.

Presenting a Complete Solution

I've worked out the limits as x approaches both positive and negative infinity, and clearly calculated that the limit as x tends to positive infinity is 1. Conversely, as x tends towards negative infinity, I've confirmed that the limit is -1. I have ensured a detailed explanation of the steps. I'm ready to present the solution, including the vital steps and highlighting the differences between the two cases. Finally, I will suggest to show a graph of the function to visually confirm the results.

The limit as $x \rightarrow \infty$ is **1**. The second box is likely asking for the limit as $x \rightarrow -\infty$, which is **-1**.

The strategy for finding limits at infinity of rational-like functions is to divide the numerator and the denominator by the highest power of x found in the denominator.

Case 1: Limit as $x \rightarrow \infty$

1. **Identify the highest power.** The highest power of x in the denominator is inside the root: x^6 . The effective power is therefore $\sqrt{x^6} = x^3$.
2. **Divide by the highest power.** Divide the numerator and the denominator by x^3 .

$$\lim_{x \rightarrow \infty} \frac{\frac{x^3}{x^3}}{\frac{\sqrt{x^6+9}}{x^3}}$$

3. **Simplify.** To bring x^3 inside the square root, we square it, making it x^6 .

$$\lim_{x \rightarrow \infty} \frac{1}{\sqrt{\frac{x^6+9}{x^6}}} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{9}{x^6}}}$$

4. **Evaluate.** As $x \rightarrow \infty$, the term $\frac{9}{x^6}$ goes to 0.

$$\frac{1}{\sqrt{1+0}} = \frac{1}{1} = 1$$

Case 2: Limit as $x \rightarrow -\infty$

There is one crucial difference when x approaches negative infinity.

1. **The square root rule.** The term $\sqrt{x^6}$ is technically equal to $|x^3|$. When x is a negative number, x^3 is also negative, so $|x^3| = -x^3$.
2. **Divide by the highest power.** When we divide the denominator by x^3 , we get:

$$\frac{\sqrt{x^6+9}}{x^3} = \frac{|x^3|}{x^3} \sqrt{1 + \frac{9}{x^6}}$$

Since $x \rightarrow -\infty$, $\frac{|x^3|}{x^3} = -1$.

3. **Evaluate.** This introduces a negative sign into our final calculation.

$$\lim_{x \rightarrow -\infty} \frac{1}{-\sqrt{1 + \frac{9}{x^6}}} = \frac{1}{-\sqrt{1+0}} = -1$$

Would you like me to generate a graph of this function? It provides a very clear visual of why there are two different horizontal asymptotes (one at $y=1$ and the other at $y=-1$).