

3. As $x \rightarrow \infty$, the term $\frac{9}{x^6}$ goes to 0, leaving:

$$\frac{1}{\sqrt{1+0}} = 1$$

4. For the limit as $x \rightarrow -\infty$, there's a small but crucial difference. When x is negative, x^3 is also negative. So, $\sqrt{x^6} = |x^3| = -x^3$. This introduces a negative sign in the calculation, making the result **-1**.

Would you like me to create a quick reference table summarizing which techniques (like direct substitution, factoring, conjugates) to use for different types of limit problems?



8. Evaluate the limit using the appropriate Limit Law(s). (If an answer does not exist, enter DNE.)

$$\lim_{x \rightarrow 2} \sqrt{\frac{3x^2 + 4}{3x - 2}}$$



Thinking:

Examining the Limit

I've begun analyzing the limit. This appears simpler than previous ones, as it's a direct substitution case. I'll substitute $x=2$ to see if it yields a determinate answer. No need to look for any special tricks, but I will check for any unusual behaviour.

Revising Direct Substitution



Direct substitution yielded a fraction with a non-zero denominator. The numerator is positive, and the fraction's value, after calculation, will be positive. The square root of this value will then be the solution.

Applying Direct Substitution

Direct substitution led to a numerator of 16 and a denominator of 4, both positive. Their quotient is 4, leading to a valid square root solution of 2. It was a straightforward application of limit laws. I will now create an answer structure for the user, with both a concise answer and a thorough explanation.

Calculating the Limit

I've completed the calculation and the limit is 2. The function is continuous at $x=2$, allowing for direct substitution. Upon substituting and simplifying, I found the fraction under the square root equals 4, the square root of which is 2. I now consider providing a follow up example to the user.

The value of the limit is **2**.

This limit can be found using **direct substitution**. According to the Limit Laws, if a function is continuous at the point you're approaching, you can find the limit by simply plugging that value into the function.

Step-by-Step Calculation

1. Substitute the value Plug $x = 2$ into the expression inside the square root:

$$\frac{3(2)^2 + 4}{3(2) - 2}$$

2. Simplify the expression

$$\frac{3(4) + 4}{6 - 2} = \frac{12 + 4}{4} = \frac{16}{4} = 4$$

Since the denominators not zero and the value inside the root is positive, direct substitution is valid.

3. Evaluate the final limit Now, apply the square root to the result:

$$\sqrt{4} = 2$$

Would you like to see an example of a similar-looking problem where direct substitution *doesn't* work, to understand what to do in that case?



9. Find the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \rightarrow \infty} \left(\sqrt{64x^2 + x} - 8x \right)$$



Thinking:

Reviewing Limit Solution

I'm revisiting the limit problem, aiming to streamline my approach. I've re-examined my earlier solution, focusing on clarity and conciseness, especially the rationalization step and simplification. I'm confident in the final answer but want to refine the explanation for optimal readability.

Refining the Explanation

I'm now presenting the limit solution. I've reevaluated the solution, ensuring the steps are clear, including the rationalization and simplification. The result is the

final value of $\frac{1}{16}$. I begin by checking for an indeterminate form, then apply the conjugate, simplify, and divide to arrive at the solution.

Calculating the Limit

I've completed the detailed calculation of the limit, confirming the answer is $\frac{1}{16}$. I've clearly laid out each step: checking for the indeterminate form, rationalizing with the conjugate, simplifying the expression, and finally, evaluating the limit as x approaches infinity by dividing numerator and denominator by x .

Revisiting Infinity Limit

I've re-examined the limit problem, providing a detailed solution. I began by confirming the indeterminate form, then applied the conjugate. Simplifying and dividing by x led me to the final answer of $\frac{1}{16}$. I'm confident in my explanation but could extend the problem. Would you like to see how to solve the limit as x approaches negative infinity? The process is very similar but has one important trick.

The limit is **** $\frac{1}{16}$ ****.

Trying to evaluate this limit by just plugging in infinity results in an **indeterminate form** ($\infty - \infty$), which doesn't give us the answer. The standard technique for this type of problem is to multiply by the **conjugate**.

Step-by-Step Solution

1. Multiply by the Conjugate The conjugate of $(\sqrt{64x^2 + x} - 8x)$ is $(\sqrt{64x^2 + x} + 8x)$. We multiply the expression by a fraction equal to 1, using the conjugate:

$$\lim_{x \rightarrow \infty} (\sqrt{64x^2 + x} - 8x) \cdot \frac{\sqrt{64x^2 + x} + 8x}{\sqrt{64x^2 + x} + 8x}$$

2. Simplify the Numerator This creates a difference of squares

$(a - b)(a + b) = a^2 - b^2$ in the numerator:

$$\left(\sqrt{64x^2 + x}\right)^2 - (8x)^2 = (64x^2 + x) - 64x^2 = x$$

The limit expression is now:

$$\lim_{x \rightarrow \infty} \frac{x}{\sqrt{64x^2 + x} + 8x}$$

3. Divide by the Highest Power To solve this limit at infinity, we divide every term in the numerator and denominator by the highest power of x in the denominator. Here, the highest power is x (or $\sqrt{x^2}$).

$$\lim_{x \rightarrow \infty} \frac{\frac{x}{x}}{\frac{\sqrt{64x^2 + x}}{x} + \frac{8x}{x}} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{\frac{64x^2 + x}{x^2}} + 8}$$

4. Evaluate the Limit Simplify the fraction inside the square root and take the limit. As $x \rightarrow \infty$, the term $\frac{1}{x}$ approaches 0.

$$\lim_{x \rightarrow \infty} \frac{1}{\sqrt{64 + \frac{1}{x}} + 8} = \frac{1}{\sqrt{64 + 0} + 8} = \frac{1}{8 + 8} = \frac{1}{16}$$

Would you like to see how to solve the limit as x approaches negative infinity? The process is very similar but has one important trick.



Where did $1/x$ come from?



Thinking:

Analyzing the Origin



I am now delving into the user's specific query regarding the previous limit problem. My focus has shifted to clarifying the origin of the $1/x$ term within the equation. It seems they need a more detailed explanation of that particular step in the process.

Refining the Explanation

I'm focusing now on clarifying the crucial step where the $1/x$ emerges. My aim is to provide a comprehensive breakdown of the division and simplification process within the square root, making it undeniably clear. The user's query highlights the need for a more granular explanation of the algebraic manipulation.

Clarifying the Derivation

I've been working on breaking down the process to show how the $\frac{1}{x}$ term arises. I've broken down each step to make the logic evident. The key is in bringing the x inside the square root. I'm hoping to get a "yes, that's it" or a "can you explain more" response.

Clarifying the Source

I'm now detailing the step-by-step process where the $\frac{1}{x}$ term emerges. I'm focusing on ensuring absolute clarity. I'll outline the division of the square root by x and the subsequent simplification. This involves bringing the x into the square root by squaring it. My goal is to leave no room for ambiguity. I will present this information concisely, making sure the step is well-defined.

The $\frac{1}{x}$ came from the process of dividing the entire denominator, $\sqrt{64x^2 + x} + 8x$, by x .

Here's a closer look at the algebra for the tricky part, which is dividing the square root term by x .

Step-by-Step Breakdown

1. **Isolate the term.** We start with the fraction from that specific part of the denominator:

$$\frac{\sqrt{64x^2 + x}}{x}$$

2. **Bring 'x' inside the root.** To combine everything under one square root, you have to square the x in the denominator as you bring it inside.

$$\sqrt{\frac{64x^2 + x}{x^2}}$$

3. **Split the fraction.** Now you can divide each term in the numerator by the new denominator, x^2 .

$$\sqrt{\frac{64x^2}{x^2} + \frac{x}{x^2}}$$

4. **Simplify.** This gives you the final expression, which contains the $\frac{1}{x}$ term.

$$\sqrt{64 + \frac{1}{x}}$$

Does that step-by-step breakdown of the algebra make sense?



Why did it become 0?



Thinking:

Clarifying the Limit

I'm focusing now on the limit, specifically why it equates to zero. I've re-examined the denominator as x grows infinitely large, noting the interplay between the fraction within the square root and the constant. It's becoming

clearer how the shrinking term impacts the overall expression's ultimate value, which I'm confident is 0.

Reiterating the Core

I'm now focusing on the fundamental concept: how a denominator's increasing size shrinks a fraction's value toward zero. I plan to reinforce this with a pizza analogy, followed by numerical examples like $x = 10$, 1000, and 1,000,000, illustrating how $1/x$ approaches zero.

Confirming Zero's Arrival

I'm solidifying the rationale for zero. It's the core of the proof. As the denominator of $\frac{1}{x}$ expands, its value shrinks toward zero. The pizza analogy further clarifies the point, with numerical examples supporting the notion. Should I advance to discuss scenarios with different powers in the numerator and denominator?

Solidifying the Fraction's Behavior

As x grows infinitely large, the fraction's value shrinks toward zero. Imagine sharing a pizza: the more friends, the smaller each slice. With x as 10, the fraction is 0.1; with x as 1000, it's 0.001. Now, would you like to explore limits where the denominator doesn't dominate?

That's a great question, it hits on the key idea of limits at infinity.

The term $\frac{1}{x}$ approaches 0 because as the variable x gets incredibly large, you are dividing 1 by a massive number. The result is a number that is extremely close to zero.

Think of it like sharing a single pizza 🍕:

- If you share it with 10 people ($x = 10$), each person gets $\frac{1}{10}$ (or 0.1) of the pizza.
- If you share it with 1,000 people ($x = 1,000$), each person gets a tiny crumb, $\frac{1}{1000}$ (or 0.001) of the pizza.

If you share it with a billion people ($x = 1,000,000,000$), each person's "slice" is practically nothing—it's so close to zero that for the purpose of the limit, we say it **is** zero.

In calculus, this is a fundamental rule: for any constant number 'c', the limit of $\frac{c}{x}$ as x approaches infinity is always 0.

Does this idea of a fraction getting infinitely small make sense?



So if it's infinity it's 0?



Thinking:

Clarifying Infinity's Nuance

I'm now focusing on clarifying the user's misconception. The response "So if it's infinity it's 0?" is an oversimplification. I plan to gently correct this by highlighting the distinction between limits approaching infinity and the limit being zero. I want to build their understanding, not just correct them.

Revising the Correction Strategy

I'm now revising my strategy for correcting the user's misconception. While I still plan to acknowledge their partially correct understanding, my focus has shifted towards providing a clearer counterexample to illustrate the specific conditions under which the limit becomes zero. The goal is to build a more solid foundation for their understanding of limits.

Refining the Explanation

I'm now zeroing in on a precise explanation, starting with a direct clarification: a constant divided by an infinitely large number does approach zero. The key is



emphasizing "approaches" rather than simply "is". I'll use the counterexamples to reinforce this distinction. I'll then proactively guide the user to expand their understanding of limit behavior.

Deconstructing the Rule's Nuances

I'm now refining the explanation to ensure clarity. I'll begin by acknowledging the correct part of the user's assumption, followed by a specific explanation. The goal is to provide a more targeted counterexample to show that the generalization "infinity equals zero" isn't universally valid. I will then propose the next step, building a solid path for a deeper understanding of limit rules.

Yes, but with one crucial detail: the limit is **0** when you have a **constant number** divided by **something going to infinity**.

Your statement is correct for cases like:

- $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$
- $\lim_{x \rightarrow \infty} \frac{150}{x^2} = 0$

However, a limit involving infinity isn't always 0. For example, if infinity isn't in the denominator, the result is different:

- $\lim_{x \rightarrow \infty} x = \infty$

So the key is that infinity must be in the **denominator** for the fraction's value to shrink to zero.

The rule we've been discussing is what happens when the highest power of x is in the denominator. Would you like to go over the rules for the other two main scenarios for limits at infinity?

