



6. Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \rightarrow 9} \frac{3 - \sqrt{x}}{9x - x^2}$$

Thinking:

Exploring the Conjugate

I'm now focusing on the conjugate method to tackle this limit. I see that multiplying the numerator and denominator by the conjugate of the numerator will likely simplify the expression and eliminate the indeterminate form. This strategy appears promising, and I'm eager to execute it.

Executing the Conjugate Strategy

I've successfully applied the conjugate, transforming the initial limit. Simplifying the numerator yielded $(9 - x)$, perfectly setting up the next crucial step. The denominator is now $x(9 - x)(3 + \sqrt{x})$. It's clear that the $(9 - x)$ terms will cancel, which removes the indeterminate form. This transformation is pivotal to finding the solution.

Implementing the Solution

The solution using the conjugate method is now complete. I've worked through each step, starting with multiplying by the conjugate and culminating in the final answer of $1/54$. The process involved simplification and cancellation of the indeterminate form. This direct approach yielded a clear path to the solution. I believe I have fulfilled your request thoroughly.



Finalizing the Solution

I've laid out the complete solution using the conjugate method, confirming the limit's value is $1/54$. The step-by-step process is clearly detailed, from identifying the conjugate to arriving at the final answer. I'm now ready to understand your feedback on the method. I'm curious which of the two methods (factoring or using the conjugate first) you found more straightforward.

Of course. The limit is $\frac{1}{54}$.

Here's the step-by-step solution using the conjugate method you asked for.

Solving by Multiplying by the Conjugate

1. Identify the Conjugate The part of the expression with the square root is the numerator, $3 - \sqrt{x}$. Its conjugate is $3 + \sqrt{x}$.

2. Multiply the Numerator and Denominator We multiply the entire fraction by $\frac{3 + \sqrt{x}}{3 + \sqrt{x}}$:

$$\lim_{x \rightarrow 9} \frac{3 - \sqrt{x}}{9x - x^2} \cdot \frac{3 + \sqrt{x}}{3 + \sqrt{x}}$$

3. Simplify the Expression

• **Numerator:** Multiplying the conjugates gives a difference of squares:

$$(3 - \sqrt{x})(3 + \sqrt{x}) = (3)^2 - (\sqrt{x})^2 = 9 - x$$

• **Denominator:** We now have: $(9x - x^2)(3 + \sqrt{x})$

This leaves us with the new limit expression:

$$\lim_{x \rightarrow 9} \frac{9 - x}{(9x - x^2)(3 + \sqrt{x})}$$

4. Factor and Cancel Now, notice we can factor the $9x - x^2$ term in the denominator by pulling out an x :

$\lim_{x \rightarrow 9} \frac{9-x}{x(9-x)(3+\sqrt{x})}$ The $(9-x)$ terms on the top and bottom cancel out, leaving: $\lim_{x \rightarrow 9} \frac{1}{x(3+\sqrt{x})}$

5. Evaluate the Limit Finally, we can substitute $x = 9$ into the simplified expression:

$$\frac{1}{9(3+\sqrt{9})} = \frac{1}{9(3+3)} = \frac{1}{9(6)} = \frac{1}{54}$$

Now that you've seen this problem solved by factoring first and by using the conjugate first, did one method make more sense to you than the other?



7. Given that

$$\lim_{x \rightarrow 2} f(x) = 1 \quad \lim_{x \rightarrow 2} g(x) = -5 \quad \lim_{x \rightarrow 2} h(x) = 0,$$

find the limits, if they exist. (If an answer does not exist, enter DNE.)

(a) $\lim_{x \rightarrow 2} [f(x) + 2g(x)]$

(b) $\lim_{x \rightarrow 2} [g(x)]^3$

(c) $\lim_{x \rightarrow 2} \sqrt{f(x)}$

(d) $\lim_{x \rightarrow 2} \frac{5f(x)}{g(x)}$

(e) $\lim_{x \rightarrow 2} \frac{g(x)}{h(x)}$

(f) $\lim_{x \rightarrow 2} \frac{g(x)h(x)}{f(x)}$

Thinking:

Analyzing Limit Properties

I'm currently focused on dissecting the problem, which involves applying limit properties to evaluate several expressions. So far, I've identified the core challenge: using given limit values of $f(x)$, $g(x)$, and $h(x)$ to compute others. I'm strategizing how to best leverage the limit laws for sums, differences, products, and quotients. I'll need to carefully consider the impact of constants and different function combinations.

Solving Limit Expressions



I've made headway, specifically solving the first four parts. Using the properties of limits, I calculated the limits of sums, constant multiples, powers, roots, and quotients. So far, the results are: -9 for (a), -125 for (b), 1 for (c), and -1 for (d). I've confirmed that I'm correctly applying limit laws and paying close attention to any zero denominators.

Concluding Limit Calculations

I've successfully solved parts (a) through (d), resulting in -9, -125, 1, and -1, respectively. My focus now is on parts (e) and (f), where I'm applying the quotient and product rules. I've determined that the limit in (e) does not exist because the denominator approaches zero while the numerator approaches a non-zero value. Now I'm working on part (f), aiming to complete the entire problem.

Evaluating Limits Thoroughly

My latest focus has been on evaluating the limits in parts (e) and (f). For (e), I've determined the limit DNE due to division by zero. I've concluded that the limit in (f) is 0, using the product and quotient rules. I am now confident that I have completely and correctly solved the problem.

Here are the answers:

- **(a) -9*** (Sum Law, Constant Multiple Law)
- **(b) -125*** (Power Law)
- **(c) 1*** (Root Law)
- **(d) -1*** (Quotient Law, Constant Multiple Law)
- **(e) DNE (Does Not Exist)*** (Quotient Law; Division by zero in the denominator causes the function to approach infinity)
- **(f) 0*** (Product Law, Quotient Law)

Would you like a review of the underlying Limit Laws that made these calculations possible?

Reviewing Problem Completion

I've carefully considered the given information, reviewed my previous findings, and finalized the solutions. My process involved applying the limit laws for sums, differences, products, quotients, and powers to each part of the problem. After a thorough analysis, I can confirm that all the limits have been successfully evaluated.

Absolutely. You can solve these by applying the basic **Limit Laws** to the given values.

Here are the answers:

- (a) **-9**
 - (b) **-125**
 - (c) **1**
 - (d) **-1**
 - (e) **DNE** (Does Not Exist)
 - (f) **0**
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Explanations

You're essentially just substituting the given limit values into the expressions.

(a) Sum and Constant Multiple Law $\lim_{x \rightarrow 2} [f(x) + 2g(x)] = \lim_{x \rightarrow 2} f(x) + 2 \cdot \lim_{x \rightarrow 2} g(x) = 1 + 2(-5) = -9$

(b) Power Law $\lim_{x \rightarrow 2} [g(x)]^3 = [\lim_{x \rightarrow 2} g(x)]^3 = (-5)^3 = -125$

(c) Root Law $\lim_{x \rightarrow 2} \sqrt{f(x)} = \sqrt{\lim_{x \rightarrow 2} f(x)} = \sqrt{1} = 1$

(d) Quotient Law $\lim_{x \rightarrow 2} \frac{5f(x)}{g(x)} = \frac{5 \cdot \lim_{x \rightarrow 2} f(x)}{\lim_{x \rightarrow 2} g(x)} = \frac{5(1)}{-5} = -1$

(e) Quotient Law (Special Case) $\lim_{x \rightarrow 2} \frac{g(x)}{h(x)} = \frac{\lim_{x \rightarrow 2} g(x)}{\lim_{x \rightarrow 2} h(x)} = \frac{-5}{0}$ Because the denominator approaches **0** while the numerator approaches a non-zero number **(-5)**, the limit **does not exist (DNE)**.

(f) Product and Quotient Law $\lim_{x \rightarrow 2} \frac{g(x)h(x)}{f(x)} = \frac{(\lim_{x \rightarrow 2} g(x))(\lim_{x \rightarrow 2} h(x))}{\lim_{x \rightarrow 2} f(x)} =$

$$\frac{(-5)(0)}{1} = \frac{0}{1} = 0$$

Would you like a brief summary of the main Limit Laws used here, like the Sum, Product, and Quotient Rules?



8. Evaluate the limit using the appropriate Limit Law(s). (If an answer does not exist, enter DNE.)

$$\lim_{x \rightarrow 2} \sqrt{\frac{3x^2 + 4}{3x - 2}}$$

9. Find the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \rightarrow 8} \left(\sqrt{64x^3 + x} - 8x \right)$$

10. Find the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \rightarrow \infty} \frac{x^3}{\sqrt{x^6 + 9}}$$



Thinking:

Calculating the Limit



I've begun analyzing the provided image and, as requested, am focusing on problem 8. Initial assessment reveals it's a limit of a root, which simplifies things.

Approaching Problem 9

Having tackled problem 8, I'm now transitioning to problem 9, a limit at infinity. The initial substitution yields an indeterminate form. My strategy shifts to multiplying the expression by its conjugate to simplify and potentially resolve the limit. The conjugate of $\sqrt{64x^2 + x} - 8x$ is $\sqrt{64x^2 + x} + 8x$.

Calculating Solutions

After solving the first two limits, I've moved onto the last problem. This one is looking trickier, as it involves several concepts. I am focused on the best approach to find the final solution.

Solving the Problems

I've completed problems 8 and 9. Problem 8 was straightforward, involving a direct substitution of $x = 2$ followed by evaluation of a cube root. Problem 9 required the use of the conjugate and clever manipulation to evaluate the limit at infinity, which came out to $1/16$. I'm now analyzing problem 10, recognizing it also involves a limit at infinity.

Completing the Limits

I've successfully solved all three limit problems. For the final problem, I determined the limit to be 1. I also considered the case as x approaches negative infinity, correctly identifying and addressing the absolute value implications, resulting in the same answer. I'm confident in the solutions now.

Calculating the Solutions

I've completed the analysis of the three limit problems. For problem 8, I found the answer by direct substitution. For problem 9, the answer was obtained using the conjugate. In problem 10, the answer was 1, also evaluated by direct

substitution. However, I decided to check what happens at negative infinity, which gave me -1 .

Evaluating the Solutions

I've completed a thorough analysis of all three limit problems. Problem 8 was solved through direct substitution, resulting in $\sqrt{3}$. For problem 9, using the conjugate yielded a limit of $\frac{1}{16}$. In the final problem, I considered both x approaching positive and negative infinity, finding the limit as x approaches infinity is 1 and as x approaches negative infinity it's -1 . I have ensured the answers are accurate.

Here are the solutions to the limit problems.

8. The limit is $\sqrt{3}$.
9. The limit is $\frac{1}{16}$.
10. The limit as $x \rightarrow \infty$ is 1 . The second box is likely for the limit as $x \rightarrow -\infty$, which is -1 .

Explanations

Problem 8

$$\lim_{x \rightarrow 2} \sqrt{3}$$

For this problem, we can use **direct substitution** because the function is continuous at $x = 2$ (the denominator is not zero).

1. Plug $x = 2$ into the expression inside the cube root:

$$\frac{3(2)^2 + 4}{3(2) - 2} = \frac{3(4) + 4}{6 - 2} = \frac{16}{4} = 4$$

2. Take the cube root of the result:

$$\sqrt{3}$$

Problem 9

$$\lim_{x \rightarrow \infty} (\sqrt{64x^2 + x} - 8x)$$

This is an indeterminate form ($\infty - \infty$), so we need to multiply by the **conjugate**.

1. The conjugate of $\sqrt{64x^2 + x} - 8x$ is $\sqrt{64x^2 + x} + 8x$.

2. Multiply the expression by $\frac{\text{conjugate}}{\text{conjugate}}$:

$$\lim_{x \rightarrow \infty} \frac{(\sqrt{64x^2 + x} - 8x)(\sqrt{64x^2 + x} + 8x)}{\sqrt{64x^2 + x} + 8x}$$

3. Simplify the numerator (it's a difference of squares):

$$(64x^2 + x) - (8x)^2 = 64x^2 + x - 64x^2 = x$$

4. The limit is now $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{64x^2 + x} + 8x}$. To solve this, divide every term by the highest power of x in the denominator, which is x (or $\sqrt{x^2}$).

$$\lim_{x \rightarrow \infty} \frac{\frac{x}{x}}{\sqrt{\frac{64x^2}{x^2} + \frac{x}{x^2} + \frac{8x}{x}}} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{64 + \frac{1}{x} + 8}}$$

5. As $x \rightarrow \infty$, the term $\frac{1}{x}$ approaches 0.

$$\frac{1}{\sqrt{64 + 0 + 8}} = \frac{1}{8 + 8} = \frac{1}{16}$$

Problem 10

$$\lim_{x \rightarrow \infty} \frac{x^3}{\sqrt{x^6 + 9}}$$

For limits at infinity, we divide the numerator and denominator by the highest power of x in the denominator.

1. The highest power in the denominator is effectively $\sqrt{x^6} = x^3$.

2. Divide the top and bottom by x^3 :

$$\lim_{x \rightarrow \infty} \frac{\frac{x^3}{x^3}}{\frac{\sqrt{x^6 + 9}}{x^3}} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{\frac{x^6 + 9}{x^6}}} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{9}{x^6}}}$$

