$g: \mathbb{R}^2 \to \mathbb{R}$

$g(x,y,z) = 3x^2 + 4xy + 8xz + 4yz + 3z^2$

$A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$ $\quad C = \{e_1, e_2, e_3\}$ base canonica

$B = \{v_1, v_2, v_3\}$ $v_1 = (1,0,0), v_2 = (1,1,0)$ $v_3 = (0,1,1)$

$M = P^t A P$ $\quad P = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ $\quad P^T = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ $\quad M =$

$Q_A(\lambda) = -(1+\lambda)(\lambda+8)(\lambda+1)$

$\begin{vmatrix} 3-\lambda & 2 & 4 \\ 2 & -\lambda & 2 \\ 4 & 2 & 3-\lambda \end{vmatrix} \underset{C_1-C_3}{=} \begin{vmatrix} -1-\lambda & 2 & 4 \\ 0 & -\lambda & 2 \\ 1+\lambda & 2 & 3-\lambda \end{vmatrix} \underset{F_1+F_3}{=} \begin{vmatrix} 0 & 4 & 7-\lambda \\ 0 & -\lambda & 2 \\ 1+\lambda & 2 & 3-\lambda \end{vmatrix} = (1+\lambda)\begin{vmatrix} 4 & 7-\lambda \\ -\lambda & 2 \end{vmatrix} =$

$= (1+\lambda)(8+7\lambda-\lambda^2)$ $\quad \dfrac{-7 \pm \sqrt{49+32}}{-2}$ $\quad \dfrac{7 \pm 9}{2} \begin{cases} 8 \\ -1 \end{cases}$

VAPS	ma
-1	2
$+8$	1

Diagonalitza

$\mathrm{Ker}\,(A-\lambda I)$

$\langle (1,-2,0), (0,-2,1) \rangle$

$\langle (2,1,2) \rangle$

$\lambda = -1$

$\mathrm{Ker}\,(A+I)$

$\left(\begin{array}{ccc|c} 4 & 2 & 4 & 0 \\ \cancel{2} & \cancel{1} & \cancel{2} & \cancel{0} \\ \cancel{4} & \cancel{2} & \cancel{4} & \cancel{0} \end{array}\right)$ $\quad$ Sol $= y\ \dfrac{-4x-4z}{2} = -2x-2z$ $\quad (x, -2x-2z, z) = x(1,-2,0) + z(0,-2,1)$

$\lambda = 8$ $\quad \mathrm{Ker}\,(A-8I)$

$\left(\begin{array}{ccc|c} -5 & 2 & 4 & 0 \\ 2 & -8 & 2 & 0 \\ 4 & 2 & -5 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -5 & 2 & 4 & 0 \\ 0 & -36 & 18 & 0 \\ 0 & 18 & -9 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} -5 & 2 & 4 & 0 \\ 0 & -2 & 1 & 0 \end{array}\right)$ $\quad$ Sol $z = 2y$

$-5x + 2y + 8y = 0$ $\quad x = 2y$

$(2y, y, 2y) = y(2,1,2)$

$u_1 = (1,-2,0)$ $\quad u_2 = (0,-2,1)$, $\quad u_3 = (2,1,3)$ $\quad$ formen una base de VEPS (no necessariament ortogonals)

u_1, u_2', u_3 base ortogonal on $u_2' = u_2 - \dfrac{\phi(u_1,u_2)}{\phi(u_2,u_2)} u_1$

$\phi(u_1,u_2) = (1,-2,0)\begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}\begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} = (1,-2,0)\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} = -4$

$\phi(u_2,u_2) = (0,-2,1)\begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}\begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} = -5$ $\quad u_2' = (0,-2,1) - \dfrac{4}{5}(1,-2,0) = (0,-2,1) - \left(\dfrac{4}{5}, \dfrac{-8}{5}, 0\right)$

$(-1,2,0)\begin{pmatrix} -\frac{4}{5} \\ -\frac{2}{5} \\ 1 \end{pmatrix} = 0 \Rightarrow$ son ortogonals $\quad = \left(-\dfrac{4}{5}, -\dfrac{2}{5}, 1\right)$

per a que surtin els vaps a la diagonal dividim ~~el vector~~ per el seu modul

$u_1 = (1,-2,0)$ $\quad u_2' = \left(-\dfrac{4}{5}, -\dfrac{2}{5}, 1\right)$ $\quad u_3 = (2,1,2)$

$\Downarrow \div \sqrt{1^2+2^2}$ $\quad \Downarrow \div \sqrt{(\frac{4}{5})^2+(\frac{2}{5})^2+1^2}$ $\quad \Downarrow \div \sqrt{2^2+1^2+2^2}$

$w_1 = \left(\dfrac{1}{\sqrt{5}}, \dfrac{-2}{\sqrt{5}}, 0\right)$ $\quad w_2 = \left(-\dfrac{4\sqrt{5}}{15}, -\dfrac{2\sqrt{5}}{15}, \dfrac{\sqrt{5}}{3}\right)$ $\quad w_3 = \left(\dfrac{2}{3}, \dfrac{1}{3}, \dfrac{2}{3}\right)$

$D = P^t \cdot A \cdot P$

$D = P^t \cdot \Delta \cdot P$

$$D = P^t \cdot \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{5}} & -4\sqrt{5} & \frac{2}{3} \\ \frac{-2}{\sqrt{5}} & \frac{-2\sqrt{5}}{15} & \frac{1}{3} \\ 0 & \frac{\sqrt{5}}{3} & \frac{2}{3} \end{pmatrix} =$$

per fer ortonormal agafar la ortogonal i dividir per la norma $= \sqrt{q(u_i)}$

$\div \sqrt{q u_1} \Downarrow$ $\quad \div \sqrt{q u_2} \Downarrow$ $\quad \div \sqrt{q u_3}$

$\bar{u}_1 =$ $\quad \bar{u}_2 =$ $\quad \bar{u}_3 =$

per a que hi hagui una base ortonormal la forma quadratica te que ser definida positiva, sinó no existeix la norma dels vectors

$\sqrt{-x}$ no existeix

$$\begin{aligned}\dot{x} &= 7x - 5y + e^{-3t} \\ \dot{y} &= 10x - 8y - e^{-3t}\end{aligned} \Big\} \quad \begin{matrix} x(t) \\ y(t) \end{matrix}$$

sist no homogeni

on $F(t) = \begin{pmatrix} e^{-3t} \\ -e^{-3t} \end{pmatrix}$

<u>Sistema homogeni</u>

$$\begin{aligned}\dot{x} &= 7x - 5y \\ \dot{y} &= 10x - 8y\end{aligned} \Big\} \quad \begin{matrix} x^{(h)} \\ y^{(h)} \end{matrix}$$

$\Downarrow$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \underbrace{\begin{pmatrix} 7 & -5 \\ 10 & -8 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \end{pmatrix}$$

$A = \begin{pmatrix} 7 & -5 \\ 10 & -8 \end{pmatrix}$ $\quad Q_A(\lambda) = (2-\lambda)(-3-\lambda)$

$D = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$

VAPS	ma	Ker $(A-\lambda I)$
$\lambda_1 = 2$	1	$\langle (1,1) \rangle$
$\lambda_2 = -3$	1	$\langle (1,2) \rangle$

$P = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

<u>canvi de variables</u>

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = P^{-1} \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = D \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \Rightarrow$$

<u>Solució</u> $\quad$ el VAP

$$\begin{aligned} z_1(h) &= c_1 \cdot e^{2t} \\ z_2(h) &= c_2 \cdot e^{-3t} \end{aligned}$$

Desfem el canvi

$$\begin{pmatrix} x \\ y \end{pmatrix} = P \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} c_1 e^{2t} \\ c_2 e^{-3t} \end{pmatrix}$$

$$\boxed{\begin{aligned} x(h) &= c_1 e^{2t} + c_2 e^{-3t} \\ y(h) &= c_1 e^{2t} \quad 2c_2 e^{-3t} \end{aligned}}$$

Solució particular

$$F(t) = \begin{pmatrix} e^{-3t} \\ -e^{-3t} \end{pmatrix}$$

$$\begin{pmatrix} x(p) \\ y(p) \end{pmatrix} = B(t) \cdot \int B^{-1}(t) \cdot F(t)\, dt$$

on $\begin{pmatrix} x(h) \\ y(h) \end{pmatrix} = \underbrace{\begin{pmatrix} e^{2t} & e^{-3t} \\ e^{2t} & 2e^{-3t} \end{pmatrix}}_{B(t)} \begin{pmatrix} c1 \\ c2 \end{pmatrix}$

$\det B(t) = e^{-t}$

$$\text{adj}(B(t)) = \begin{pmatrix} 2e^{-3t} & -e^{2t} \\ -e^{-3t} & e^{2t} \end{pmatrix}$$

$$B^{-1}(t) = \begin{pmatrix} 2e^{-2t} & -e^{-2t} \\ -e^{3t} & e^{3t} \end{pmatrix}$$

$$\text{adj}(B(t))^t = \begin{pmatrix} 2e^{-3t} & -e^{-3t} \\ -e^{2t} & e^{2t} \end{pmatrix}$$

$$B^{-1}(t) \cdot F(t) = \begin{pmatrix} 2e^{-2t} & -e^{-2t} \\ -e^{3t} & e^{3t} \end{pmatrix} \begin{pmatrix} e^{-3t} \\ -e^{-3t} \end{pmatrix} = \begin{pmatrix} 3e^{-5t} \\ -2 \end{pmatrix} \underset{\text{integrar}}{\Rightarrow} \begin{pmatrix} -\frac{3}{5} e^{-5t} \\ -2t \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x(p) \\ y(p) \end{pmatrix} = \begin{pmatrix} e^{2t} & e^{-3t} \\ e^{2t} & 2e^{-3t} \end{pmatrix} \begin{pmatrix} -\frac{3}{5} e^{-5t} \\ -2t \end{pmatrix} =$$

$$x(p) = -\frac{3}{5} e^{-3t} - 2t \cdot e^{-3t} = \left(-\frac{3}{5} + 2t\right) e^{-3t}$$

$$y(p) = \frac{-3}{5} e^{-3t} - 4t e^{-3t} = \left(\frac{-3}{5} - 4t\right) e^{-3t}$$

Solució general

$$\boxed{\begin{aligned} x &= c1 e^{2t} + c2 e^{-3t} + \left(\frac{-3}{5} + 2t\right) e^{-3t} \\ y &= c1 e^{2t} + 2c2 e^{-3t} + \left(\frac{-3}{5} - 4t\right) e^{-3t} \end{aligned}}$$

$S = \{ (x,y,z) \in \mathbb{R}^3 \mid x + ay - az = 0, \quad x + y - z = 0 \}$

$T = \langle (3,2,5), (3, a-1, a+2) \rangle$

T

A matriu de vectors

$A^t = \begin{pmatrix} 3 & 2 & 5 \\ 3 & a-1 & a+2 \end{pmatrix} \sim \begin{pmatrix} 3 & 2 & 5 \\ 0 & a-3 & a-3 \end{pmatrix}$

$\boxed{\text{Si } a \neq +3}$ Rang $A = 2 = \dim T \Rightarrow$ i una base $\{(3,2,5), (0, a-3, a-3)\}$

$\boxed{\text{Si } a = 3}$

Rang $A = 1 = \dim T \Rightarrow$ i una base $\{(3,2,5)\}$

~~S~~

$\left.\begin{matrix} x + ay - az = 0 \\ x + y - z = 0 \end{matrix}\right\}$ $\left(\begin{matrix} 1 & a & -a \\ 1 & 1 & -1 \end{matrix}\middle| \begin{matrix} 0 \\ 0 \end{matrix}\right) \sim \left(\begin{matrix} 1 & a & -a \\ 0 & 1-a & -1+a \end{matrix}\middle| \begin{matrix} 0 \\ 0 \end{matrix}\right)$

Si $a = 1$

$x + y - z = 0 \Rightarrow x = z - y \Rightarrow (z-y, y, z) = y(-1,1,0)\ z(1,0,1)$

$S = \langle (-1,1,0), (1,0,1) \rangle \Rightarrow \dim S = 2$

i una base

Si $a \neq 1$

$\left.\begin{matrix} x + ay - az = 0 \\ (1-a)y + (-1+a)z = 0 \end{matrix}\right\}$ $\begin{matrix} \Rightarrow x = 0 \\ y = z \end{matrix}$ $\Rightarrow (0, z, z) = z(0,1,1)$ dim $S = 1$ una base $(0,1,1)$

$S = T$ — si $a = 3$ $\dim S = \dim T = 1$ $\quad S = \langle (0,1,1) \rangle$ i $T = \langle (3,2,5) \rangle$

linealment ind.

$(3,2,5) \notin S \Rightarrow$ no són iguals $T \neq S$

$F \subseteq G$ veure si els vectors de la base són de G

Si $a = 1$ $\quad \dim S = \dim T = 2$

$S = (-1,1,0), (1,0,1) \rangle$

~~$x = 0$ ... $x + y - z = 0$~~

$T = \langle (3\,2\,5), (0,-2,-2) \rangle$

$(3\,2\,5)$ es satisfà equacions

$(0,-2,-2)$ es satisfà equacions

$\Rightarrow T \subseteq S \Rightarrow T = S$ quant $a = 1$

$\underline{a=0}$

$S = \langle (0,1,1) \rangle$

$T = \langle (3,2,5)\ \cancel{(0,-3,-3)} \rangle (0,1,1) \rangle$

$\underline{S+T} = \langle (0,1,1), (3,2,5) \rangle \cancel{(0,1,1)}$ dim $S+T = 2$; una base $\{(0,1,1),(3,2,4)\}$

$\underline{S \cap T}$ $\underset{2}{\dim S+T} = \underset{1}{\dim S} + \underset{2}{\dim T} - \underset{1}{\dim S \cap T}$

$= \{ (x,y,z) \in \mathbb{R}^3$

$(x,y,z) \in S \Rightarrow \begin{cases} x=0 \\ x+y-z=0 \end{cases}$

$(x,y,z) \in T \Rightarrow \text{Rang} \begin{pmatrix} 3 & 0 & x \\ 2 & 1 & y \\ 5 & 1 & z \end{pmatrix} = 2$

$\begin{pmatrix} 3 & 0 & x \\ 2 & 1 & y \\ 5 & 1 & z \end{pmatrix} = \begin{pmatrix} 3 & 0 & x \\ 0 & 3 & 3y-2x \\ 0 & 3 & 3z-5x \end{pmatrix} \sim \left| \begin{matrix} 3 & 0 & x \\ 0 & 3 & 3y-2x \\ 0 & 0 & 3z-3x-3y \end{matrix} \right| \Rightarrow 3z-3x-3y=0$

$\begin{matrix} x=0 \to \\ x+y-z=0 \to \\ +3z-3x-3y=0 \end{matrix} \left(\begin{matrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ -3 & -3 & +3 \end{matrix} \middle| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right) \sim \left(\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ \cancel{0} & \cancel{-3} & \cancel{+3} \end{matrix} \middle| \begin{matrix} 0 \\ 0 \\ \cancel{0} \end{matrix} \right)$

Sol

$x = 0$

$y = z$

$\Downarrow$

$(0,z,z) = z\,(0,1,1)$

$= \{ (x,y,z) \in \mathbb{R}^3$

mots que pujan el rang

$\begin{pmatrix} 3 & 3 & x \\ 2 & a-1 & y \\ 5 & a+2 & z \end{pmatrix} = \begin{pmatrix} 3 & 3 & x \\ 0 & 3a-9 & 3y-2x \\ 0 & 3a-9 & 3z-5x \end{pmatrix} \sim \left| \begin{matrix} 3 & 3 & x \\ 0 & 3a-9 & 3y-2x \\ 0 & 0 & -3x-3y+3z \end{matrix} \right| = 0$

$T = \{ (x,y,z)\ \mathbb{R}^3, \boxed{x+y-z=0}$ si $a \neq 3$

si $a=3$ $T = \{ (x,y,z)\ \mathbb{R}^3$ $\boxed{\begin{matrix} 3y-2x=0 \\ x+y-z=0 \end{matrix}}$